Technical Comments

Comment on "Effect of Nonlinear Prebuckling State on the Postbuckling Behavior and Imperfection-Sensitivity of Elastic Structures"

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COHEN[1] has presented a general development of an initial postbuckling analysis which takes into account nonlinear prebuckling deformations of an elastic structure. He reports a discrepancy between several of his results and those obtained in a similar analysis by Fitch.[2] As briefly as possible, we will resketch this analysis, following the same notation as that used by Cohen, to show that Fitch's results are the correct ones. The reader is referred to these two papers for definitions and details which will not be repeated here.

The discrepancy between the two sets of results arises in the analysis of the effect of a small imperfection on the buckling load of the structure. If the imperfection is denoted by $\bar{\xi}\bar{u}$ where $\bar{\xi}$ is the imperfection amplitude, then the strain resulting from an additional displacement u is

$$\epsilon = L_1(u) + \tfrac{1}{2}L_2(u) + \bar{\xi}L_{11}(u,\bar{u}) \qquad (1)$$

As Cohen asserts, the displacement can be written as

$$u = u_0 + \xi u_1 + \xi^2 u_2 + \xi^3 u_3 + \ldots \qquad (2)$$

since to the order to which the analysis is carried, terms such as $\bar{\xi}\xi u_{11}$, etc. do not enter. Thus, the strain can be written as

$$\epsilon = \epsilon_0 + \bar{\xi}L_{11}(\bar{u},u_0) + \xi[L_1(u_1) + L_{11}(u_0,u_1)] + \xi^2[L_1(u_2) + \tfrac{1}{2}L_2(u_1) + L_{11}(u_0,u_2)] + \xi^3[\quad] + \xi\bar{\xi}[\quad] + \ldots \qquad (3)$$

where $\epsilon_0 \equiv L_1(u_0) + \frac{1}{2}L_2(u_0)$ is the strain arising from the prebuckling deformation of the perfect structure u_0. The variation in the strain can also be written as

$$\delta\epsilon = L_1(\delta u) + L_{11}(u_0,\delta u) + \bar{\xi}L_{11}(\bar{u},\delta u) + \xi L_{11}(u_1,\delta u) + \xi^2 L_{11}(u_2,\delta u) + \ldots \qquad (4)$$

Now, these expansions are substituted into the principle of virtual work (here we will consider only dead loading)

$$\sigma\cdot\delta\epsilon = q\cdot\delta u \qquad (5)$$

Here the stress-strain relation $\sigma = H(\epsilon)$ is used along with the fact that the prebuckling deformation, itself, satisfies

$$\sigma_0\cdot\delta\epsilon_0 = q\cdot\delta u$$

where $\delta\epsilon_0 \equiv L_1(\delta u) + L_{11}(u_0,\delta u)$. A nonlinear algebraic equation relating ξ, $\bar{\xi}$, and the load parameter is obtained by taking $\delta u = u_1$ and by using the equations for u_1, u_2, etc. listed by Fitch and Cohen. This is similar to what has been done by Cohen[1]; but the term $\bar{\xi}L_{11}(\bar{u},u_0)$ in the expression for ϵ seems to have been omitted in that reference. Thus, for example, the term given on p. 1619 of Ref. 1 for augmenting the right-hand side of Eq. (6c) is incorrect.

The final results obtained are in the form of Cohen's Eqs. (33 and 35–38) but with corrected expressions for α and β. In particular, we find

$$\alpha = [\sigma_0{}^*\cdot L_{11}(\bar{u},u_1) + \sigma_1\cdot L_{11}(\bar{u},u_0{}^*)]/[\lambda_C\sigma_0{}^{(1)*}\cdot L_2(u_1) + 2\lambda_C\sigma_1\cdot L_{11}(u_0{}^{(1)*},u_1)]$$

with a similar type of expression, if the live loading term q_1 is taken into account. When the imperfection is in the shape of the buckling mode (i.e., $\bar{u} = u_1$), this reduces to

$$\alpha = [\sigma_0{}^*\cdot L_2(u_1) + \sigma_1\cdot L_{11}(u_1,u_0{}^*)]/[\lambda_C\sigma_0{}^{(1)*}\cdot L_2(u_1) + 2\lambda_C\sigma_1\cdot L_{11}(u_0{}^{(1)*},u_1)]$$

or, alternatively, this can be written in the form equivalent to that given by Fitch,

$$\alpha = -\sigma_1\cdot L_1(u_1)/[\lambda_C\sigma_0{}^{(1)*}\cdot L_2(u_1) + 2\lambda_C\sigma_1\cdot L_{11}(u_0{}^{(1)*},u_1)]$$

The discussion given at the end of Ref. 1 concerning the effect of imperfections other than those in the shape of the buckling mode must be changed accordingly to account for the correct expression for α.

References

[1] Cohen, G. A., "Effect of Nonlinear Prebuckling State on the Postbuckling Behavior and Imperfection-Sensitivity of Elastic Structures," *AIAA Journal*, Vol. 6, No. 8, Aug. 1968, pp. 1616–1619.

[2] Fitch, J. R., "The Buckling and Post-Buckling Behavior of Spherical Caps under Concentrated Load," *International Journal of Solids Structures*, Vol. 4, 1968, pp. 421–446.

Received September 1968.

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Reply by Author to J. R. Fitch and J. W. Hutchinson

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THE author would like to thank J. R. Fitch and J. W. Hutchinson for correcting the error in the subject paper. The error arose as a result of applying the third of Eqs. (5) to the imperfect structure, whereas only the first of Eqs. (5) remains valid for this case.

As a result of this correction, the following equations of the subject paper should be changed as shown:

$$\alpha = [\sigma_0{}^*\cdot L_{11}(\bar{u},u_1) + \sigma_1\cdot L_{11}(\bar{u},u_0{}^*) - \lambda_c q_1(\bar{u})\cdot u_1]/\lambda_c F^{(1)}(u_1,u_1) \qquad (34a)$$

$$\beta = \{\sigma_0{}^{(1)*}\cdot L_{11}(\bar{u},u_1) + \sigma_1\cdot L_{11}(\bar{u},u_0{}^{(1)*}) - q_1(\bar{u})\cdot u_1 + H[L_{11}(u_0{}^{(1)*},u_1)]\cdot L_{11}(\bar{u},u_0{}^*) - \alpha\lambda_c H[L_{11}(u_0{}^{(1)*},u_1)]\cdot L_{11}(u_0{}^{(1)*},u_1) - \tfrac{1}{2}\alpha\lambda_c F^{(2)}(u_1,u_1)\}/F^{(1)}(u_1,u_1) \qquad (34b)$$

Received March 24, 1969.

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